Atmospheric Pressure and Refractive Indices, with a Corresponding Table of Indices of Optical Glass.

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(Communicated by Sir Joseph Petavel, F.R.S. Received October 28, 1921.)

A new method of obtaining refractive index was described in 1902.*

Instead of depending on the accuracy of arbitrary measurements of refracting angle or the accuracy of construction of a prism, this method was based on the mean value of the three angles of a triangle. The description of this method was accompanied by a Table of the principal refractive indices of quartz, calcite, and fluorite, obtained in this manner, and was followed later by similar Tables of the refractive indices of vitreous silica, water, and optical glass,† and in February, 1915, these were in turn followed by a Table of the refraction temperature coefficients of optical glass.‡

Since the error due to the method itself amounted to less than unity in the seventh figure, there was promise of great accuracy. But, nevertheless, with every care, and in spite of the correction for temperature, errors even exceeding unity in the fifth decimal place were frequently manifest, requiring repeated and tedious observation to eliminate. Hence suspicion fell on the barometer. But it was pointed out by the late Prof. Sylvanus Thompson, F.R.S., and afterwards by Sir Arthur Schuster, who indicated the lines to be worked on, that the modulus of rigidity of glass precluded its being sensibly affected by atmospheric pressure,‡ and that, therefore, any such effect must be due to the refraction of air alone.

It was evident that such influence was not altogether to be neglected, and a recent application of barometer readings as a correction for the variation of the refractive index of glass under pressure has made it clear that such effects exceed unity in the fifth decimal place and may therefore have significant value.

Description and Application of the Correction.

Formulæ for the variation of the refractive index of air with temperature at given pressure have been given independently by MM. Benoit and Mascart, also by von Lang.§

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* 'Roy. Soc. Proc.,' vol. 70, p. 329.

† 'Roy. Soc. Proc.,' A, vol. 73, p. 201; vol. 78, p. 406; vol. 87, p. 189.

‡ 'Roy. Soc. Proc.,' A, vol. 91, p. 319.

§ Dufet, 'Données Numériques,' vol. 1, p. 78.

The mean of the constants given by Benoit and Mascart is 0·00374, and this has been adopted. It gives results which agree well with those obtained by von Lang's formula.*

Let t = temperature recorded at observation.

n = refractive index of air for the mean D line.†

Then $n_0 = 1{\cdot}0002923$ at standard pressure (760 mm.).

$$n_t - 1 = \frac{n_0 - 1}{1 + 0{\cdot}00374t}.$$

The application of these formulæ for our purpose is as follows.

Let

$$\left.\begin{array}{l} P = \text{standard pressure} \\ p = \text{observed} \quad ,, \end{array}\right\} \text{in mm. of mercury,}$$

n = refractive index of air at standard pressure and observed temperature,

$1 + \frac{(n-1)p}{P} = n'$ = refractive index of air at observed pressure and observed temperature,

μ = refractive index of glass at observed pressure and observed temperature,

$\frac{\mu n'}{n} = \mu'$ = refractive index of glass at standard pressure and observed temperature.

From this and two measurements of refractive index of the same wave-length, but at different temperatures, the refraction temperature coefficient at standard pressure for 1° C. may be determined.

Using this coefficient as a final correction, indices for other wave-lengths at standard pressure and observed temperature may be brought to standard pressure and temperature (15° C.).

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[*] Benoit's figures in the above formula are $0{\cdot}00366t$, Mascart's $0{\cdot}00382t$; von Lang's formula is

$$n_t = n_0 - 0{\cdot}0^6905t + 0{\cdot}0^8835t^2 \ (loc.\ cit.).$$

[†] The variation of the refractive index of air with difference of wave-length has been measured by Kaiser and Runge and given by Dufet (*loc. cit.*); it is small and may here be neglected.

Worked Example.

Obsn.

(1) Refractive index at 13·5° C. and 735 mm. bar. = 1·552267.

(2) „ „ 20·0° C. „ 769 „ = 1·552264.

$$(1)\ n = 1{\cdot}0002783. \qquad (2)\ n = 1{\cdot}0002720.$$

$$1+\frac{(n-1)p}{\mathrm{P}} = n' = 1{\cdot}0002691. \qquad n' = 1{\cdot}0002752.$$

$$(\text{At } 13{\cdot}5^\circ\ \text{C})\ \mu = 1{\cdot}552267. \qquad (\text{At } 20^\circ\ \text{C.})\ \mu = 1{\cdot}552264.$$

$$\frac{\mu n'}{n} = \mu' = 1{\cdot}552253. \qquad \mu' = 1{\cdot}552269.$$

$$1{\cdot}552269 - 1{\cdot}552253 = 0{\cdot}000016. \qquad 20{\cdot}0^\circ - 13{\cdot}5^\circ = 6{\cdot}5.$$

Hence temp. ref. coefft. at standard pressure $= \dfrac{0{\cdot}000016}{6{\cdot}5} = 0{\cdot}00000246$ for 1° C., and the refractive index at standard pressure and 15° C. = 1·552257.

As already described, the refractive indices for each wave-length required are corrected similarly for standard pressure and observed temperature, and are then brought to 15° C. by using the refraction temperature coefficient.

It must be stated here that whereas the refractive indices in the before-mentioned papers and in all previous papers were given at a temperature of 15° C., using a correction for temperature common to glass and air, it is only in the following indices that the correction for atmospheric pressure has been applied in addition. All indices, including those previously given are for glass in air, not *in vacuo*.

The refractive indices of a melting of fluor crown, whose temperature refraction coefficient was a minus quantity, have already been given.*

In the fluor crown the indices of which are given above, this coefficient has changed sign, having apparently rounded the corner. In this connection it is perhaps interesting to recall the fact that the temperature refraction coefficients of fluorite, quartz, and vitreous silica all have the minus sign.

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* 'Roy. Soc. Proc.,' vol. 91, p. 319.

Refractive Indices of Glasses at Standard Pressure and 15° C., in Ascending Order of their Dispersions.

Designation*	Fluor Crown. D. 591. Type 337.	Borosilicate Crown. C. 3537. Type 646.	Hard Crown. C. 4497. Type 9663.	Dense Flint. C. 3673. Type 1034.	Dense Flint. C. 3467. Type 3743.	Dense Flint. C. 1794. Type 572.	Dense Flint. C. 4380. Type 3607.
$F-C = \delta\mu$	0·007330.	0·00797$\bar{9}$.	0·00866$\bar{9}$.	0·01603$\bar{0}$.	0·016484.	0·016975.	0·017166.
Probable error (1)...	±0·000007.	±0·000014.	±0·000009.	±0·000023.	±0·000012.	±0·000011.	±0·000072.
Probable error (2)†	±0·000008.	±0·000015.	±0·000009.	±0·000022.	±0·000012.	±0·000011.	±0·000075.
$\nu = \mu - 1/\delta\mu$	67·4$\bar{2}$.	63·7$\bar{1}$.	59·95.	37·7$\bar{2}$.	37·05.	36·46.	36·29.
Wave-lengths.							
A′ 7682 K_α	1·489322	1·503146	1·51416$\bar{7}$	1·59482$\bar{0}$	1·600663	1·608662	1·61247$\bar{6}$
B′ 7066 He	1·49072$\bar{2}$	1·50456$\bar{1}$	1·515673	1·59740$\bar{5}$	1·603293	1·611369	1·61518$\bar{6}$
6708 Li	1·49162$\bar{4}$	1·50552$\bar{5}$	1·516720	1·599193	1·60511$\bar{8}$	1·61324$\bar{7}$	1·617103
C. 6563 H_α	1·49201$\bar{4}$	1·505939	1·517162	1·599996	1·60597$\bar{0}$	1·61412$\bar{5}$	1·618015
D. 5893 Na	1·49420$\bar{1}$	1·508325	1·519725	1·604584	1·610659	1·618983	1·62291$\bar{2}$
Λ. 5607 Pb	1·49535$\bar{3}$	1·509548	1·521086	1·607060	1·613209	1·621593	1·625566
5461 Hg	1·49603$\bar{4}$	1·51029$\bar{1}$	1·52186$\bar{6}$	1·608536	1·614713	1·623147	1·62714$\bar{1}$
E. 5270 Fe	1·496963	1·511313	1·522998	1·610622	1·61688$\bar{0}$	1·62538$\bar{3}$	1·629405
F. 4861 H_β	1·49934$\bar{4}$	1·513918	1·525831	1·61602$\bar{6}$	1·62245$\bar{4}$	1·63110$\bar{0}$	1·635181
Φ. 4678 Cd_6	1·500591	1·515280	1·527375	1·619011	1·62553$\bar{9}$	1·634261	1·63842$\bar{5}$
4415 Cd_7	1·50267$\bar{5}$	1·51762$\bar{5}$	1·52993$\bar{2}$	1·624094	1·63075$\bar{9}$	1·63964$\bar{7}$	1·64391$\bar{9}$
G′ 4341 H_γ	1·50337$\bar{8}$	1·518358	1·53072$\bar{3}$	1·62571$\bar{7}$	1·63249$\bar{4}$	1·64138$\bar{4}$	1·645683
4046 Hg	1·506379	1·521723	1·534446	1·63340$\bar{1}$	1·64037$\bar{5}$	1·64950$\bar{2}$	1·65398$\bar{4}$

* C = Chance Bros. and Co., Ltd. D = Derby Crown Glass Co., Ltd.

† (1) 'Roy. Soc. Proc.,' vol. 70, p. 332. (2) 'Roy. Soc. Proc.,' A, vol. 87, p. 189.

Note.—In the above table of refractive indices much accuracy is not claimed for the 6th decimal place, which, however, serves to determine the last unit in the 5th. The probable error is that of all the glass available, in each case, three different prisms having been taken at hazard from different parts of the melting ('Roy. Soc. Proc.,' A, vol. 87, p. 190).

Table of Refraction Coefficients for 1° C. in each Melting.

Designation.	Temperature refraction coefficients.
Fluor Crown, D. 591	0·000000$\bar{5}$
Borosilicate Crown, C. 3537	0·0000004
Hard Crown, C. 4497	0·0000024
Dense Flint, C. 3673	0·000005$\bar{0}$
„ „ C. 3467	0·000004$\bar{0}$
„ „ C. 1794	0·0000062
„ „ C. 4380	0·000006$\bar{2}$

Examples of the Application of Corrections for Temperature and Pressure.

Line.	Barometer.	Temperature.	Crude index.	Corrected for temperature.	Corrected for temperature and pressure.
	mm.	° C.			
A	758·3	14·375	1·489187	1·489185	1·489184
„	736·5	14·954	1·489195	1·489195	1·489182
F	756·2	12·639	1·631101	1·631116	1·631114
„	748·7	14·653	1·631122	1·631124	1·631117

Note.—Each pair of the above are for the same line and melting, but at different temperatures and pressures.

(*Note Added December* 8, 1921.)

Errors due to Eccentricity of the Goniometer Circle.

Since all refractive indices are practically affected alike by them, these errors are not of first importance, but may perhaps rank next to those due to atmospheric pressure.

The correction I have adopted is based on the formula for probable error (1) already referred to,* but in this case the three "group deviations" are not taken for the formula a. Instead three new groups are built up by adding together all deviations measured, respectively, when 0 of the circle is made zero, when 120 is made zero, and when 240 is made zero, no matter at what angle of the prism, provided that each angle is always represented in all three groups.

The values of those which show the greatest difference are now taken from it for x and y as before. Our formula then gives us the probable value of the eccentricity error in terms of the deviations, but we know not whether it is a *plus* or *minus* error.

The circle to which reference has been made is an 8-inch circle by

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* 'Roy. Soc. Proc.,' A, vol. 87, p. 190.

Troughton. Another circle by Cooke, with a diameter of 18 inches, mounted on an instrument with 3-inch O.G., has been used throughout for check purposes, and also for experimental work on the refractive indices of water and of sea-water,[*] and this circle is the more correct of the two.[†] Now this larger circle always gives a higher reading than the small one. Presumably then the error of the smaller circle must be additive.

The mean result in the index of some three hundred observations taken in this way amounted to 0·0000092, while the errors of observation taken over the same ground amounted to 0·00000085.

Taking these also as additive one to the other, we have a total error of 0·000010 for addition to the indices at the discretion of the user.

The advantage claimed for this method over any method which determines the total error due to the circle is, that that portion of the circle alone which has been put into use is put under contribution.

A complete measurement of index with all the observations, including the "group deviations" as ordinarily taken, is given in 'Roy. Soc. Proc.,' vol. 70, p. 338.

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[*] 'Roy. Soc. Proc.,' A, vol. 78.

[†] *Ibid.*, p. 408.